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The Wiener Polynomial for the Subdivision Graphs

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Abstract: One of the oldest distance-based topological index, the Wiener index is studied, and expressions for the Wiener polynomial of the subdivision graph of the Tadpole graph $T_{n,k}$, the cycle C_n, the wheel graph W_{n+1} and the Helm graph H_{n+1} are presented in this paper.

Keywords: Wiener polynomial; subdivision graph; Tadpole graph; wheel graph; Helm graph

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1 Introduction

Mathematical calculations are absolutely necessary to explore important concepts in chemistry. Mathematical chemistry is a branch of theoretical chemistry for prediction and discussion of the molecular structure using mathematical methods without necessarily referring to quantum mechanics. Chemical graph theory is an important tool for studying molecular structures.

Let G be an undirected graph without multiple edges and loops, the vertex-set and edge-set of which are represented by $V(G)$ and $E(G)$, respectively. If x and y are two vertices of G, then $d(x,y)$ denotes the length of a minimum length of the path connecting x and y. A distance counting polynomial introduced by Hosoya is $W(G,q)=\sum d(G,k)q^k$, which is known as the Hosoya polynomial[10] or the Wiener polynomial. The Wiener index[9] of a graph G, named after the chemist Harold Wiener, who considered it in connection with paraffin boiling points, is given by $W(G)=\sum d_G(x,y)$, where d_G denotes the distance in G. Also

$$W(G)=\frac{\mathrm{d}[W(G,q)]}{\mathrm{d}\,q}\Big|_{q=1}.$$

The subdivision graph $S(G)$[4,8,7] is obtained from G by replacing each of its edge by a path of length two, or equivalently, by inserting an additional vertex into each edge of G. The $T_{n,k}$ Tadpole graph[5] is the graph obtained by joining a cycle graph C_n to a path of length k.

The wheel graph W_{n+1} is defined as the graph K_1+C_n, where K_1 is the singleton graph and C_n is the cycle graph[3]. The Helm graph H_{n+1} is the graph obtained from the wheel graph W_{n+1}, by adjoining a pendent edge at each node of the cycle[6]. For all terminologies and notations which are not defined in this paper, we refer to Harary[2].

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Biography: Ranjini P. S (Born in 1978), Female, Senior Lecturer. Research field: graph theory.

In this paper, we concentrate on a systematic study and obtained an expression for the Wiener polynomial of the subdivision graphs of Tadpole graphs and wheel graphs. Here, we studied the subdivision graph of $T_{n,k}$ and the Wiener polynomials of the graph $S(T_{3,k})$ and $S(T_{4,k})$. Extended the studies further to obtain explicit expressions for the Wiener polynomial of the subdivision graph of the cycle C_n, the wheel graph W_{n+1} and the Helm graph H_{n+1}.

2 Wiener polynomial of subdivision graphs

In this section, we derived an expression for the Wiener polynomials of the subdivision graph of Tadpole graphs, wheels, Helm graphs and cycles.

Theorem 1 The Wiener polynomial of $S(T_{3,k})$ is

$$W\big(S(T_{3,k}),q\big)=q^n+3q^{n-1}+5q^{n-2}+6q^{n-3}+(e-1)q^3+eq^2+eq,$$

where $n=2k+3$ and $e=6+2k$.

Proof The result would be proved by the method of induction on k. Consider the graph $S(T_{3,1})$, the number of edges in the graph is 8, the vertex $v_{1'}$ is of maximum distance 5 and the vertices v_1, v_2 and $v_{1'}$ are of distance 4 with respect to v_4, v_4 and $v_{4'}$, respectively, where v_4 is the pendent vertex in the path and $v_{4'}$ is the neighbor of v_4 in $S(T_{3,1})$. The seven combination of vertices contributes for the distance 3 and 8 combination of vertices contributes the distance 2 and 1, respectively. Hence the Wiener polynomial of $S(T_{3,1})$ is

$$W\big(S(T_{3,1}),q\big)=q^5+3q^4+7q^3+8q^2+8q.$$

If $k=2$, consider the graph $S(T_{3,2})$, add one edge e to $S(T_{3,1})$ and do the subdivision of the edge e then obtained $S(T_{3,2})$. The subdivision of one edge will increase the path length by 2. So the distance from $v_{1'}$ to $v_{5'}$ is increased by 2 in $S(T_{3,2})$ where $v_{5'}$ is the neighbor of v_4 in $S(T_{3,2})$. So the distance is 7. The vertices v_1 and v_2 in $S(T_{3,1})$ which are at a distance 4 is now at a distance 6 in $S(T_{3,2})$. The vertex which contribute to the distance 7 in $S(T_{3,1})$ will also contribute to a distance 6 with respect to the vertex $v_{5'}$. Hence the three vertices contribute the distance 6. The same vertices which induce the distance 6 along with two more vertices contribute for the distance 5 and so on. Also, there will be an increase in the coefficient of q^3, q^2 and q by two in the Wiener polynomial of $S(T_{3,1})$. Hence the Wiener polynomial of $S(T_{3,2})$ is

$$W\big(S(T_{3,2}),q\big)=q^7+3q^6+5q^5+6q^4+9q^3+10q^2+10q.$$

Assume that the result is true for $S(T_{3,k})$. The number of edges in $S(T_{3,k})$ is $6+2k=e$ and the maximum distance is $2k+3$, say n. So the Wiener polynomial is

$$W\big(S(T_{n,2}),q\big)=q^n+3q^{n-1}+5q^{n-2}+6q^{n-3}+\cdots+(e-1)q^3+eq^2+eq.$$

The result is true for $S(T_{3,k+1})$. The number of edges is $e+2=8+2k$ and the maximum distance is $n+2$. So replace n with $n+2$ and e with $e+2$ in the above, obtain the Wiener polynomial of $S(T_{3,k+1})$ as

$$W\big(S(T_{3,k+1}),q\big)=q^{n+2}+3q^{n+1}+5q^n+6q^{n-1}+\cdots+(e-1)q^3+eq^2+eq.$$

Hence the Wiener polynomial of $S(T_{3,k})$ is

$$W\big(S(T_{3,k}),q\big) = q^n + 3q^{n-1} + 5q^{n-2} + 6q^{n-3} + \cdots + (e-1)q^3 + eq^2 + eq,$$

where $n = 2k+3$ and $e = 6+2k$.

Theorem 2 The Wiener polynomial for $S(T_{4,k})$ is

$$W\big(S(T_{4,k}),q\big) = q^n + 3q^{n-1} + 5q^{n-2} + 7q^{n-3} + 8q^{n-4} + \cdots + (e-2)q^4 + eq^3 + eq^2 + eq,$$

where $n = 2k+4$ and $e = 2k+8$.

Proof We prove this by mathematical induction. The number of edges in $S(T_{4,1})$ is 10 and the maximum distance is 6. The vertex v_2 is of distance 6 and the vertices $v_{1'}$ and $v_{2'}$ are the neighbors of v_2 at a distance 5 with respect to the pendent vertex in the path. The 8 combination of vertices contribute for the distance 4 and the 10 combination of vertices contribute for the distance 3, 2 and 1. Hence the Wiener polynomial for $S(T_{4,1})$ is

$$W\big(S(T_{4,1}),q\big) = q^6 + 3q^5 + 8q^4 + 10q^3 + 10q^2 + 10q.$$

To show the result is true for $k = 2$. By adding one edge to $S(T_{4,1})$ and doing the subdivision, we obtain $S(T_{4,2})$. Hence the number of edges and the maximum distance will be increased by 2 with respect to $S(T_{4,1})$. Hence the maximum distance is 8 and and the number of edges is 12. The vertices which are at a distance 6 and 5 in $S(T_{4,1})$ will be at a distance 8 and 7 in $S(T_{4,2})$. The 5 vertices in the left most side of $S(T_{4,2})$ contribute for the distance 6 and the same 5 vertices along with 2 more vertices contribute for the distance 5. By increasing the coefficient of q^4, q^3, q^2 and q by 2 in $S(T_{4,1})$, then the Wiener polynomial for $S(T_{4,2})$ is

$$W\big(S(T_{4,2}),q\big) = q^8 + 3q^7 + 5q^6 + 7q^5 + 10q^4 + 12q^3 + 12q^2 + 12q.$$

The result is assumed to be true for $S(T_{4,k})$. The number of edges is $8+2k = e$, the maximum distance is $4+2k = n$. The Wiener polynomial is

$$W\big(S(T_{4,k}),q\big) = q^n + 3q^{n-1} + 5q^{n-2} + 7q^{n-3} + 8q^{n-4} + \cdots + (e-2)q^4 + eq^3 + eq^2 + eq,$$

which proves the result for $k = k+1$. The number of edges is $10+2k$ and the maximum distance is $n+2$. Replacing n with $n+2$ and e with $e+2$, then a polynomial is obtained as

$$W\big(S(T_{4,k+1}),q\big) = q^{n+2} + 3q^{n+1} + 5q^n + 7q^{n-1} + 8q^{n-2} + \cdots + eq^4 + (e+2)q^3 + (e+2)q^2 + (e+2)q.$$

Hence the Wiener polynomial for $S(T_{4,k})$ is

$$W\big(S(T_{4,k}),q\big) = q^n + 3q^{n-1} + 5q^{n-2} + 7q^{n-3} + 8q^{n-4} + \cdots + (e-2)q^4 + eq^3 + eq^2 + eq,$$

where $n = 2k+4$ and $e = 2k+8$.

Theorem 3 The Wiener polynomial for $S(C_n)$ is

$$W\big(S(C_n),q\big) = nq^n + 2nq^{n-1} + 2nq^{n-2} + \cdots + 2nq.$$

Proof The proof is made by induction on n. Let $n = 2$. The subdivision of edges of C_2 will increase the number of vertices by 2 and the number of edges also by 2. Let v_1 and v_2 be

the vertices of C_2 so that the vertices v_1 and v_2 contributes for the distance 2 and all the 4 vertices in $S(C_2)$ contribute for the distance 1.

Hence the Wiener polynomial is $W\big(S(C_2),q\big)=2q^2+4q$. Now add one more edge to C_2 and construct C_3 and do the subdivision of edges of C_3 to construct $S(C_3)$. Therefore, the number of edges in $S(C_3)$ will be increased by 2 with respect to $S(C_2)$. Hence the maximum distance is increased by 1. The 3 vertices on the left most side of $S(C_3)$, are at a distance 3 with respect to the 3 vertices on the right most side of $S(C_3)$. The 4 vertices contribute for the distance 1 in $S(C_2)$ along with 2 more vertices contribute for the distance 2. All the 6 vertices contribute for the distance 1.

Hence the Wiener polynomial is

$$W\big(S(C_3),q\big)=3q^3+6q^2+6q.$$

We assume that the result is true for $S(C_n)$. That is, the Wiener polynomial is

$$W\big(S(C_n),q\big)=nq^n+2nq^{n-1}+2nq^{n-2}+\cdots+2nq.$$

Consider $S(C_{n+1})$. Adding one edge to C_n and subdividing the edges, there is an increase by 2 in the total number of edges and the total number of vertices with respect to $S(C_n)$. Then the polynomial can be expressed as

$$W\big(S(C_{n+1}),q\big)=(n+1)q^n+2(n+1)q^{n-1}+2(n+1)q^{(n-2)}+\cdots+2(n+1)q.$$

Theorem 4 The Wiener polynomial for the subdivision graph of the wheel graph W_{n+1} is

$$W\big(S(Wn+1),q\big)=\begin{cases}\frac{1}{2}(n(3n-5))q^4+n(n+2)q^3+\frac{1}{2}(n(n+9))q^2+4nq, & \text{when } n\geq 5,\\ \frac{1}{2}(n(3n-6))q^4+n(n+2)q^3+\frac{1}{2}(n(n+9))q^2+4nq, & \text{when } n=4,\\ \frac{1}{2}(n(3n-7))q^4+n(n+2)q^3+\frac{n(n+9)}{2}q^2+4nq, & \text{when } n=3.\end{cases}$$

Proof In a wheel graph, the maximum distance is 2, hence in the subdivision graph of the wheel graph, the maximum distance is increased by two. So the maximum distance is 4. The wheel W_{n+1} contains a cycle C_n. All the n vertices which is on the subdivision of the edges in the cycle is connected to the $n-2$ vertices on the subdivision of the spokes by a path of length 4. Also all the n vertices of the wheel is connected to $n-3$ vertices by a path of length 4, for $n\geq 5$. Since every path is shared by a pair of vertices $\frac{1}{2}(n(n-3))$ vertices contributes for the distance 4. Then n paths with respect to the subdivision of the cycle is found. Hence the number of paths of length 4 is $\frac{1}{2}(n(3n-5))$. All the vertices on the subdivision of the edges on the cycle are at a distance 3 from the hub. Each vertex on the subdivision of the spoke is connected to the $n-1$ nodes of the wheel by a path of length 3. Each of the n vertices on the subdivision of the edges on the cycle is connected to one of the nodes with a distance 3. Now, find the n paths of length 3 between the nodes. Hence $n(n+2)$ paths of length 3 in $S(W_{n+1})$. All the edges on W_{n+1} lie at a distance 2 after subdivision. The n vertices on the subdivision of the edges on the cycle is connected to two of the vertices on the subdivision of the edges on the cycle with a distance 2. The vertices on the subdivision of the spoke follows permutation

concepts for contributing to a distance 2. That is the first vertex on the spoke is connected to $n-1$ vertices, the next vertex on the spoke is connected to $n-2$ vertices and so on. Hence the number of vertices at a distance 2 is $\frac{1}{2}(n(n+9))$. Hence the Wiener polynomial for $S(W_{n+1})$ is

$$W\big(S(W_{n+1}),q\big) = \frac{1}{2}\big(n(3n-5)\big)q^4 + n(n+2)q^3 + \frac{1}{2}\big(n(n+9)\big)q^2 + 4nq,$$

when $n \geq 5$.

But only for $n = 4$ and $n = 3$, then the difference in the number of vertices which are at a distance 4 is observed. Only $\frac{1}{2}n$ vertices on the subdivision of the edges on the cycle are at a distance 4 (for the case $n = 4$). This happens only because of the sharing of the edge between those pairs of edges. The coefficient of q^4 is changed to $\frac{1}{2}(n(3n-6))$.

There are no pair of vertices on the subdivision of the edges on the cycle which are at a distance 4 when $n = 3$. The coefficient of q^4 is changed to $\frac{1}{2}(3n^2 - 7n)$ for $n = 3$. Hence the Wiener polynomial of $S(W_{n+1})$ when $n = 4$ is

$$W\big(S(W_{n+1}),q\big) = \frac{1}{2}\big(n(3n-6)\big)q^4 + n(n+2)q^3 + \big(2n(n+9)\big)q^2 + 4nq.$$

For the case $n = 3$

$$W\big(S(W_{n+1}),q\big) = \frac{1}{2}\big(n(3n-7)\big)q^4 + n(n+2)q^3 + \frac{1}{2}\big(n(n+9)\big)q^2 + 4nq.$$

Theorem 5 The Wiener polynomial of the subdivision graph of the Helm graph H_{n+1} is

$$W\big(S(H_{n+1}),q\big) = \begin{cases} \frac{1}{2}(n^2-3n)q^8 + n(2n-7)q^7 + \frac{1}{2}n(5n-14)q^6 + 2n(n+1)q^5 + \frac{1}{2}(5n(n+1))q^4 \\ \quad + n(n+8)q^3 + \frac{1}{2}n(n+23)q^2 + 6nq, \quad \text{for } n \geq 6 \text{ and } n \text{ even}, \\ \frac{1}{2}(n^2-3n)q^8 + n(2n-7)q^7 + \frac{1}{2}n(5n-13)q^6 + 2n(n+1)q^5 + \frac{1}{2}(5n(n+1))q^4 \\ \quad + n(n+8)q^3 + \frac{1}{2}n(n+23)q^2 + 6nq, \quad \text{for } n \geq 6 \text{ and } n \text{ odd}, \\ \frac{1}{2}(n^2-3n)q^8 + n(2n-7)q^7 + \frac{1}{2}5n(n-3)q^6 + 2n(n+1)q^5 + \frac{1}{2}(5n(n+1))q^4 \\ \quad + n(n+8)q^3 + \frac{1}{2}n(n+23)q^2 + 6nq, \quad \text{for } n = 5, \\ \frac{1}{2}(n^2-3n)q^8 + n(2n-7)q^7 + \frac{1}{2}5n(n-3)q^6 + 2n^2q^5 + \frac{1}{2}n(5n+4)q^4 \\ \quad + n(n+8)q^3 + \frac{1}{2}n(n+23)q^2 + 6nq, \quad \text{for } n = 4, \\ nq^6 + n(2n-1)q^5 + \frac{1}{2}n(5n+1)q^4 + n(n+8)q^3 + \frac{1}{2}n(n+23)q^2 + 6nq, \\ \quad \text{for } n = 3. \end{cases}$$

Proof The graph $S(H_{n+1})$ contains the subgraph $S(W_{n+1})$ with which all the discussions made for $S(W_{n+1})$ is applicable while calculating the coefficients of q^4, q^3, q^2 and q. In the subdivision graph of the Helm graph, the maximum distance is 8 when $n \geq 4$ and is 6 when $n = 3$. Find out the number of paths of length $8, 7, 6$ etc to write the Wiener polynomial. All the pendent vertices in the Helm graph is connected to the $n-3$ pendent vertices by paths of length 8. Since it is shared by a pair of vertices, the number of paths of length 8 is

$$\frac{1}{2}n(n-3), \quad n \geq 4. \tag{1}$$

All the vertices on the subdivision of the pendent edge is connected to the $n-3$ pendent vertices by paths of length 7. So $n(n-3)$ vertices are at a distance 7 in the graph. Also the n vertices on the subdivision of the edges of the cycle are at a distance 7 with the $n-4$ pendent vertices. Hence the number of paths of length 7 is

$$n(2n-7), \quad n \geq 4. \tag{2}$$

Then there are $n(n-3)$ paths of length 6 since the n pendent vertices are connected to the $n-3$ vertices on the cycle. Find $n-3$ paths of length 6 between the subdivision vertices on the pendent edge. Since an edge is shared between a pair of vertices, there are $\frac{1}{2}n(n-3)$ paths of length 6 among these vertices. The subdivision vertices on C_n are connected to the $n-4$ vertices on the subdivision of the pendent edge by paths of length 4. So $n(n-4)$ paths of length 6 is produced by these vertices. For $n \geq 6$ and n even, there are $\frac{n}{2}$ paths of length 6 among subdivision vertices vertices of C_n and when n odd, there are n paths of length 6 among subdivision vertices vertices of C_n. Hence the number of paths of length 6 is

$$\frac{1}{2}n(5n-14), \quad \text{for } n \text{ even and } n \geq 6, \tag{3}$$

$$\frac{1}{2}n(5n-13), \quad \text{for } n \text{ odd and } n \geq 6, \tag{4}$$

$$\frac{1}{2}5n(n-3), \quad \text{for } n = 4, 5, \tag{5}$$

$$n, \quad \text{for } n = 3. \tag{6}$$

All the n pendent vertices are connected to the $n-1$ vertices on the subdivision of the spoke of the wheel by a path of length 5. Also each of the vertex on the subdivision of the pendent edge is connected to the $n-3$ vertices on C_n by a path of length 5. So $n(n-3)$ vertices are at a distance 5 in $S(H_{n+1})$ and is 0 when $n=3$. Each of the pendent vertices is connected to the two neighboring vertices on the subdivision vertices on the pendent edge by a path of length 5. Hence $2n$ paths are of length 5. We find the two paths of length 5 from the subdivision vertices of C_n by travelling a distance 3 on $S(C_n)$ and then reaching towards the pendent vertex. So $2n$ paths of this kind are of length 5. When $n=4$ and 3, this value becomes 0 since this kind of a path is already selected in $S(C_4)$ and $S(C_3)$ as per the previous goal. In $S(C_n)$, there are $2n$ paths of length 5. But it is 0 when $n=3$ and 4. So the coefficient of q^5 is

$$n(n-1) + n(n-3) + 2n + 2n + 2n = 2n(n+1), \quad n \geq 5, \tag{7}$$

$$n(n-1) + n(n-3) + 2n + 2n = 2n^2, \quad n = 4, \tag{8}$$

$$n(n-1) + n(n-3) + 2n + n = n(2n-1), \quad n = 3. \tag{9}$$

The distance between the hub and the pendent vertices is 4. All the pendent vertices are at a distance 4 with respect to the two neighboring vertices of C_n and subdivision vertices of the pendent edges are also at a distance 4 with the two neighboring subdivision vertices of the pendent edge. So $3n$ vertices are at a distance 4. The subdivision vertices of the pendent edge is connected to the $n-1$ vertices on the subdivision of the spoke by a path of length 4. So, find

the $n(n-1)$ paths of length 4. The new n subdivision vertices of C_n are connected to the two neighboring vertices on the subdivision of the pendent edge by a distance 4. But when $n=3$ it is 0. Since $S(W_{n+1})$ is a subgraph of $S(H_{n+1})$ then all the combination of vertices which are at a distance 4 in $S(W_{n+1})$. So the coefficient of q^4 is

$$3n+n(n-1)+2n+n+\frac{1}{2}\big(n(3n-5)\big)=\frac{1}{2}5n(n+1), \quad n\geq 5, \tag{10}$$

$$3n+n(n-1)+2n+n+\frac{1}{2}\big(n(3n-6)\big)=\frac{1}{2}n(5n+4), \quad n=4, \tag{11}$$

$$3n+n(n-1)+n+\frac{1}{2}\big(n(3n-5)\big)=\frac{1}{2}n(5n+1), \quad n=3. \tag{12}$$

Every pendent vertex is connected to the subdivision vertex on the spoke of the wheel and to the two neighboring vertices on $S(C_n)$ with a distance 3. All the vertices on the subdivision of the pendent edge is connected to the hub of the wheel and also to the two neighboring vertices on C_n. Hence there are $6n$ paths of length 3. The coefficient of q^3 is

$$6n+n(n+2)=n(n+8), \quad \forall\, n\geq 3. \tag{13}$$

There exists 3 paths of length 2 from every subdivision vertex on the pendent edge and one path of length 2 from every pendent vertex. Hence the coefficient of q^2 is

$$4n+\frac{1}{2}n(n+9)=\frac{1}{2}n(n+23), \quad \forall\, n\geq 3. \tag{14}$$

The number of paths of length 1 is

$$6n. \tag{15}$$

Referring to the equations corresponding to different cases, the Wiener polynomial for $S(H_{n+1})$ is thus as follows

$$W\big(S(H_{n+1}),q\big)=\begin{cases} \frac{1}{2}(n^2-3n)q^8+n(2n-7)q^7+\frac{1}{2}n(5n-14)q^6+2n(n+1)q^5+\frac{1}{2}\big(5n(n+1)\big)q^4 \\ \quad +n(n+8)q^3+\frac{1}{2}n(n+23)q^2+6nq, \quad \text{for } n\geq 6 \text{ and } n \text{ even}, \\ \frac{1}{2}(n^2-3n)q^8+n(2n-7)q^7+\frac{1}{2}n(5n-13)q^6+2n(n+1)q^5+\frac{1}{2}\big(5n(n+1)\big)q^4 \\ \quad +n(n+8)q^3+\frac{1}{2}n(n+23)q^2+6nq, \quad \text{for } n\geq 6 \text{ and } n \text{ odd}, \\ \frac{1}{2}(n^2-3n)q^8+n(2n-7)q^7+\frac{1}{2}5n(n-3)q^6+2n(n+1)q^5+\frac{1}{2}\big(5n(n+1)\big)q^4 \\ \quad +n(n+8)q^3+\frac{1}{2}n(n+23)q^2+6nq, \quad \text{for } n=5, \\ \frac{1}{2}(n^2-3n)q^8+n(2n-7)q^7+\frac{1}{2}5n(n-3)q^6+2n^2q^5+\frac{1}{2}\big(n(5n+4)\big)q^4 \\ \quad +n(n+8)q^3+\frac{1}{2}n(n+23)q^2+6nq, \quad \text{for } n=4, \\ nq^6+n(2n-1)q^5+\frac{1}{2}n(5n+1)q^4+n(n+8)q^3+\frac{1}{2}n(n+23)q^2+6nq, \\ \quad \text{for } n=3. \end{cases}$$

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细分图的Wiener多项式

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摘　要: 本文研究做为最古老的基于距离的拓朴指标之一——Wiener指标．对于Tadpole图$T_{n,k}$，循环图C_n，轮式图W_{n+1}和Helm图H_{n+1}，文中分别导出了它们的细分图的Wiener多项式的表达式．

关键词: Wiener多项式；细分图；Tadpole图；轮式图；Helm图